Study on evolving phases of accelerating generalized polygon beams

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Abstract: Recently, accelerating beam is becoming a hotspot in optics research. In this paper, we studied the evolving phases of accelerating generalized polygon beams (AGPBs) and proposed a novel method to generate this beam family. An important discovery has been made about reconstructing AGPBs only by evolving low-frequency phases in high power region, which confirms the dominant role of phase terms in the AGPBs’ evolution. We also succeeded controlling the size and quantity of AGPB’s intensity peaks in an easy and direct manner by manipulating the evolving phases in low frequency. This result not only explains the self-healing property of AGPBs but also confirms that AGPBs can be a great candidate to function as an optical tweezer to trap and free microparticles and microcreatures for certain purpose.

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References and links
1. Introduction

Accelerating beams are drawing intense interests for its unique structural traits, since Siviloglou and his team has pioneered to find the Airy beam theoretically and observed it in the innovative experiment [1,2]. Inspired by that, plenty of important work has been done to research accelerating beams with properties like power concentration, symmetry, self-bending and self-healing [3,4]. Efremidis and his associates studied the abruptly autofocusing waves and obtained a series of beam family with exotic properties [5, 9]. Some interesting research with great fancy has been done about autofocusing Airy beams carried with power-exponent-phase vortices [10]. Their work has promoted the development in optical manipulation and transport of micro-particles, living cells and micro-organisms, which is recently a hot spot in the field of bioscience [11–16]. Besides that, laser processing [17–19] and optical information transport of micro-particles, living cells and micro-organisms have benefited from the explosive development in the enrichment of accelerating beams [20].

Although so much work above has been accomplished, they all focused on the spatial intensity of accelerating beams and numerical calculation of beam phase. As phase term plays a key role in the diffracting process, it is important and necessary to experimentally study the evolving phases of accelerating beams. However, as far as we know, the evolving phase has never been able to be studied in experiment due to technical limit. This hampered the scale of people’s vision and led to a loss in significant piece of whole scientific picture. AGPB is an accelerating beam family with classical and exotic traits, such as self-bending, self-healing and energy concentrating in main lobes. With catastrophe theory, Barwick and his colleagues first studied the features and method to produce AGPBs with odd-number sides numerically [21,22]. Following that, some related work has been done to study the generation of odd-numbered AGPBs and their features. Ren and his associates studied AGPBs in experiment, and successfully discovered that the beam structure is self-reconstructed after an intensity peak is blocked by an opaque object [23,24]. However, the modulated phase form of incident beam would be an important finding in the study of AGPBs.

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beam is lengthy and not available in AGPBs with even-number sides, until we gave a simple and concise phase equation to generate all AGPBs [25]. This beam family is expected to be applied in the field of optical tweezers, transmitting signals, curving filaments and curving plasma channels. Yet, some underlying causes about AGPBs’ property like self-healing are still in mystery and remained to be resolved. What is the role of evolving phase in the diffracting AGPBs? Is there any unknown connection between evolving phase and AGPBs’ remarkable features?

In this paper, we numerically and experimentally researched the evolving phases of AGPBs based on a pixilated micropolarizer array (PMA) [26–28]. Besides that, AGPBs were successfully reconstructed only with phase parts in low frequency. Causes resulting in self-healing phenomenon have been studied and explained in experiments as well.

2. Numerical modeling

According to the diffraction integral and catastrophe polynomials, we can obtain the phase terms written in Cartesian coordinates [23,24]:

\[ \psi(x, y, z) = \text{angle}[E(x, y, z)] \]  

(1)

Where electrical field can be expressed as:

\[ E(x, y, z) = A \int_{-\infty}^{+\infty} \exp\left\{ik\left[\phi_m(\xi, \eta) - z(\xi^2 + \eta^2) + (\xi x + \eta y)\right]\right\} \times d\xi d\eta \]  

(2)

k is the wave number \(2\pi / \lambda\), A is a constant number to scale the amplitude. (\(\xi, \eta\)) means the transversal variables in input plane, which both vary from \(-1.5\) cm to \(1.5\) cm. \(\phi_m\) represents the encoded phase of the incident beam. As we have already studied the generalized phase form loaded on the incident beam to create AGPBs behind focal plane, here we directly gave out the formula in polar coordinates [25]:

\[ \phi_m(R, \theta) = CR^n \sin(m\theta + \omega) \]  

(3)
C, $\omega$ represents a constant scaling number and initial phase respectively. R means radius from image center and $\theta$ denotes the relative angle to the horizontal direction in the transversal plane. m equals to side number of APGB. In Fig. 1 from (a) to (d), we make $\omega$ to be zero and C specifically equal to $2.7 \times 10^{-2}$, $2.1 \times 10^{-2}$, $1.4 \times 10^{-2}$, $1.1 \times 10^{-2}$ while m varies from 3 to 6. These grayscale images are used as phase masks loaded on the spatial light modulator (SLM).

Based on the diffraction integral in Eqs. (1) and (2), we can numerically obtain AGPBs’ wrapped phase images and normalized intensity patterns in transversal planes during their propagation. The pixel pitch and array size of intensity images are respectively 0.007 mm and 850x850 in Figs. 2(a1)-2(a4). In Figs. 2 from (a1) to (a4), we respectively set the propagation distance behind back-focal plane to 7.5cm, 8cm, 11cm and 13cm. The focal length and wavelength of incident beam in the numerical model is determined to be 400cm and 532nm. The initial phase form of the electric field is consistent to the grayscale images in Fig. 1. After propagation in a distance, the intensity parts evolve to be polygons with different...

Fig. 2. Schematic of AGPBs’ intensity and wrapped phase in simulation. a1 to a4 represent 3D and 2D intensity patterns in traversal plane with power numbers equal to 3, 4, 5 and 6 respectively. (b1) to (b4) illustrate the corresponding wrapped phases. c1 to c4 demonstrate the processed results by wiping the phase part under power thresholds. Red circles in (c1) to (c4) represent the locations of corresponding intensity peaks. (d1) to (d4) are evolving phases of accelerating quadrangle beams at different propagating distance.
vertice numbers. At the same time, the phase parts have a similar structure in low frequency. As the AGPBs’ power concentrates inside polygon regions, it can be speculated that phase parts outside polygon centers may be practically unnecessary and meaningless to be taken into consideration. Therefore, we set a series of power thresholds to retain phase parts in high power regions while those in low parts are erased out.

\[ \Psi_m^\text{low} = L(\Psi^m, T_m) \] (4)

In Eq. (4), \( \Psi^m \) represents evolving phase of AGPBs, where \( m \) denotes the beam’s side number. \( T_m \) is threshold power value in order to silence phase parts in high frequency. Power value in Figs. 2(a1)-2(a4) lower than \( T_m \) will be regarded as low power region, and phases in this region will be wiped out. \( \Psi_m^\text{low} \) denotes the evolving low-frequency phases in high power region (LPHPR). Operator \( L \) functions to erase phase part in high frequency according to the threshold. The value of \( T_m \) we determined corresponding to \( m = 3, 4, 5 \) and 6 is equivalent to 0.05, which respectively includes 92.76%, 88.17%, 87.62% and 86.59% of beam’s total power. With such process, we obtain rotational symmetric phase patterns from (c1) to (c4) in Figs. 2. The quantity of wrapped rings inside polygon is relevant to propagating direction. The farther diffracting distance is, the more wrapped rings are there while the polygon area expands exponentially, see Figs. 2(d1)-2(d4). As self-healing property of AGPBs is proved by observation of the diffracting patterns when one intensity mainlobe is blocked, we deduced that the phase part in the central part may play an important role in reconstructing the beams.

3. Experimental setup and results

We applied the interference path in Fig. 3 to obtain the phase patterns of AGPBs. The wavelength of the laser is 532nm. Half wave plate is used to control the ratio of the reference beam and the AGPBs. After being expanded and collimated by Lens 1 and Lens 2, the source light is separated into two perpendicular linearly polarized beams through PBS1, which the S-polarized part acts as reference beam and P-polarized part is used to be modified by the phase mask imposed on the SLM. After the modified incident beam is reflected by BS and focused by Len3, two split beams are rejoined at PBS2 and occur to interfere behind the QWA. Fringe
patterns in four polarized direction are recorded by a CCD camera integrated with a PMA, which is fabricated by electron beam lithography and its units are consisted of four linear polarizers in different orientation [26–28], see illustration in Fig. 3.

The intensity of extracted four fringe patterns could be expressed as following formula:

\[
I(\alpha_i) = \frac{1}{2} \left[ I_{AGPB} + I_{REF} + 2\sqrt{I_{AGPB}I_{REF}} \cos(\varphi + 2\alpha_i) \right]
\]

\[i = 1, 2, 3, 4\]

\[\alpha_i, \alpha_2, \alpha_3, \alpha_4\] are respectively angles of four adjacent micropolarizers in the PMA which equals to \(0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\). \(I_{AGPB}\) denotes the intensity of AGPB, and \(I_{REF}\) represents the intensity of reference beam. \(\varphi\) is the sum of reference beam’s constant phase and AGPB’s phase distribution in the transversal plane behind the back focal [26]. With four fringe images and linear interpolation technique, we can obtain a wrapped phase image without sacrificing resolution.

\[
\varphi = \arctan \left( \frac{I_{155} + I_{155}}{I_0 - I_{150}} \right)
\]

When the SLM is encoded with phase masks as Fig. 1, AGPBs are generated and interfered with reference plane wave behind the backfocal plane. As is illustrated in Figs. 4(a1)-4(d1), the captured images by PMA camera are curved polygons with intensity cusps. Then, four fringes are obtained from the original images. These four fringes are corresponding to pixels in different polarization directions. After linear interpolation and computing according to Eq. (6), a raw phase image is directly acquired. As is known to us, AGPBs’ power is concentrated inside polygonal range and its vertex. In Figs. 4(a2)-4(d2), it is apparent that there exist a lot of noises inside and outside polygonal region and seems hard to distinguish phase structure from the chaos surrounding it. In the dark region around this range, it cannot occur to interfere with reference beam, which resulted in fringe noises. Therefore, we take advantage of window Fourier ridges technique to remove noises and smooth resulted structure [29], see Figs. 4(a3)-4(d3).
The filtered experimental phase structure confirms the numerical results as shown in Figs. 2(c1)-2(c4). Phase patterns are consisted of polygonal frames and wrapped rings which respectively determine the shape and height of phase. The jumped value between two neighboring rings is $2\pi$. Increasing ring numbers inside the polygonal frame indicates larger variation from the center of the most inner ring to the frame edge. To be more explicit, unwrapped phase patterns are illustrated by Figs. 4(a4)-4(d4). The grayscale represents the relative value of the total phase patterns. Three-dimensional phase feature of AGPBs seems to be polygon-arch structure, see Figs. 4(f1)-4(f4). A similar spherical-wave part plays a dominant role in the region close to the topological center, which is distorted and trapped by a polygon edge. As AGPB members have a similar phase structure property, the evolving phase
of accelerating quadrangular beam is taken as an example to study the evolution of phase pattern during its propagation. The parameter of loaded mask is the same to Fig. 1(b) except the initial phase $\omega$, which equals to $\pi$ in this situation. Illustrated by Figs. 4(e1)-4(e4), the polygonal regions are expanding and wrapped circle numbers inside this region are increasing along with the increment of propagating distance. That is to say, the gap of phase value between the topological center and polygon frame together with the area inside the frame are simultaneously increasing during the propagation.

![Diagram](image)

**Fig. 5.** Light path for generating AGPBs only by imposing evolving phase gray images without parts outside the polygonal region. BS: beam splitter. BW CCD: black-white CCD. SLM: spatial light modulator.

To prove our speculation that low-frequency phase term in high energy is a factor of crucial importance in the diffracting process, we carried out an experiment by loading the concerned phase part as phase mask and observed the evolving beam patterns at different distance. In Fig. 5, the incident beam is spitted into two parts by BS after being collimated and expanded by Lens1 and Lens 2. The transmitting part is modulated by the phase mask encoding on the phase-only SLM. Phase masks used in the lightpath are the simulated evolving phase images Figs. 2(c1) and 2(d3) as examples. The diffracting results are recorded by BW-CCD behind the back focal plane after the modulated beam is reflected by BS and focused by Lens3. An intriguing discovery was made that without phase parts in high frequency and even light intensity information, accelerating triangular and quadrangle beams are reconstructed after propagating.
AGPB’s singular point is reborn as is shown in Figs. 6(a2) and 6(b2). Diffracting distance behind the singular point from (a2) to (a5) and (b2) to (b5) are respectively 1.7 cm, 3.2 cm, 4.4 cm and 1.2 cm, 2.6 cm and 3.7 cm. Unlike the saddle-like phase form in Fig. 1, the evolving phases keep to be polygon-invariant during the process. Although the evolving phase $\Psi_{\text{low}}^{m}$ is resulted from the beam’s diffraction by the modulated phase $\phi_m$, it contains all the necessary information to recover AGPBs without the participation of corresponding intensity part. That is to say, LPHPR is a pivotal to retain beams’ polygon shapes and intensity peaks at vertexes. All the experimental results strongly support our hypothesis about the great significance of evolving phases in diffraction process. Compared to traditional technique to fulfill the generation of AGPBs, this method we proposed here is more straightforward and explicit. Beams’ topological features can be associated with LPHPR, as both of them possess a similar structure.
Inspired by the AGPBs’ reconstruction in advantage of LPHPR, another experiment was carried out to study the underlying relationship between LPHPR and AGPBs’ structure. Figures 7(m) and 7(i) is respectively the LPHPR $\Psi_{\text{low}}$ without any extra processing and its corresponding diffracting patterns. By cutting off phase components from one vertex to the center, we can obtain a series of reconstructed beam structure at the same propagating distance, see Figs. 7(e)-7(h) and 7(a)-7(d). As is demonstrated by Figs. 7(e)-7(f), it is apparent that cutting off phases at cusps has almost no influence on generating accelerating triangular beam with completed structure. However, when the eliminated part keeps going close to LPHPR’s center, the corresponding cusp was shortened and disappeared at last, see Figs. 7(c)-7(d). Followed by this result, more phase parts were erased from two vertexes to LPHPR’s core for further research. As is illustrated in Figs. 7(n)-7(p) and 7(j)-7(l), the integrity of beam’s structure has not been destroyed until phase parts nearby the center are eliminated. It can be concluded that the central part of LPHPR plays a dominant role in not only shaping the polygonality but also the intensity peaks, while the outer parts barely influenced the reborn AGPB’s structure. This conclusion confirms our surmise about the connection between LPHPR and beam’s self-healing property. Ren and his associates proved that when an intensity cusp is blocked by a small opaque object, this cusp will recover after diffracting in a distance [24]. Our experimental results can provide a reasonable explanation.
to it: when the intensity cusp is blocked, the LPHPR in central part remains to be integral and this phase part can result in the AGPB’s reconstruction after diffraction. Since intensity peaks can be generated and detached by modulating LPHPR, we can apply it to carry information in controllable-channel synchronous signal transmission. What is more, AGPBs with controllable intensity peaks are very advantageous to trap, manipulate and free nanoparticles and micro creatures in optical-tweezer setup.

4. Conclusion

In this work, we numerically and experimentally obtained evolving phase patterns of accelerating generalized polygon beams (AGPBs) with power m respectively equals to 3, 4, 5 and 6. In advantage of a CCD camera integrated with a pixilated micropolarizer array, evolving phases were acquired by phase-shifting technique in real time. Numerical and experimental results matched perfectly. Reconstruction of AGPBs only by evolving low-frequency phases in high power region (LPHPR) is fulfilled successfully in the experiment, which confirms the pivotal role of LPHPR in AGPBs’ spatial structure. What’s more, we generated AGPB with controllable intensity peaks by modified LPHPR. Compared with imposing catastrophe polynomials as phase mask, this method is able to control the intensity peaks in a more direct and easier way. Finally, AGPB’s self-healing property is analyzed and given a reasonable explanation in the experiment. Our investigation on AGPB’s LPHPR will promote a better understanding of this accelerating beam family and broaden our horizon to apply it in many fields, such as optical manipulation, filament curving and signal transmission. We also hope our research can provide a new perspective to understand self-healing property in other accelerating beams like Airy beam.

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